**Numeriske Metoder**  
Eksamen 2025

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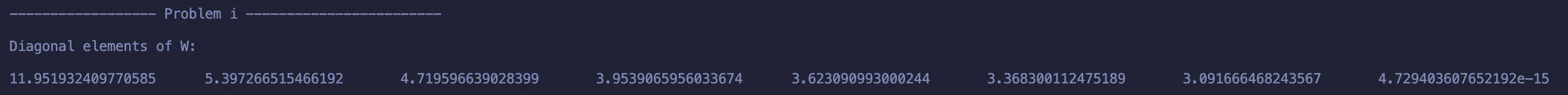
# **Exercise 1**

## **i)**

Find the Singular Value Decomposition and State the diagonal elements in W.

### **Solution**

The code reads the data file NUM S25\_Ex1A.dat for the matrix A and uses numerical recipes SVD method to print svd.w.



## **ii)**

There is a single element in W that is basically zero. Use the information from the SVD matrices to state a unit vector in the null space of A.

### **Solution**

The code uses numerical recipes SVD method svd.nullspace to find the unit vector in the null space of A. The threshold is there to find the element near zero.

A computer screen shot of a computer code

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A screen shot of a computer

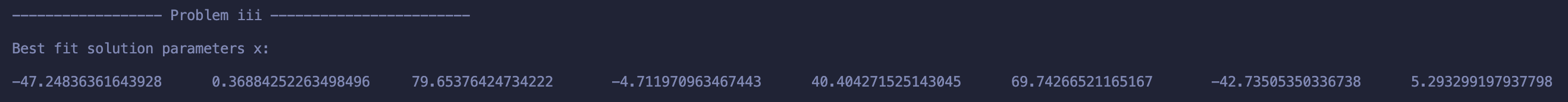
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## **iii)**

Use the Singular Value Decomposition to compute the solution x to Ax = b. State the solution x.

### **Solution**

Using SVD to find the best-fit least squares solution and returns the vector x



## **iv)**

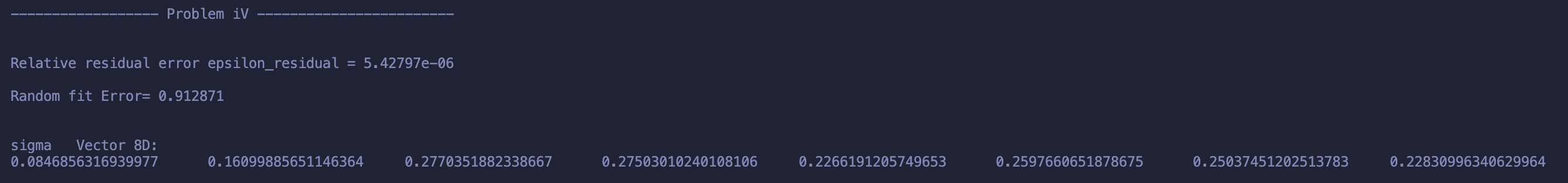
State an estimate of the accuracy on the solution x. State an explanation of how you computed the accuracy.

### **Solution**

The relative residual error was computed in the residualError() function. This measures how closely the solution satisfies the original system.

The randomFittingError() gives an estimate of the expected residual error if the data were purely random.

The sigErrorEstimate() function estimates the standard deviation sigma on each component of the solution vector x, based on the values in W.



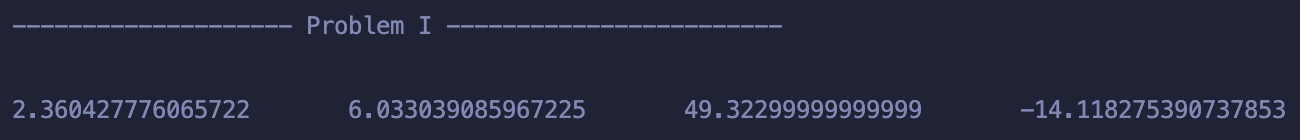
# Exercise 2

## **i)**

With state (with at least 7 digits) the values of the left-hand side of the four equations. (HINT: you should get something around

(2.36, 6.03, 49.32, −14.12).

### **Solution**



## **ii)**

Search for a solution to the equations by performing 7 iterations with the globally convergent Newton method with initial guess (x0, x1, x2, x3) = (0,0,0,0).

State for each iteration the values of x0, x1, x2, x3 and for each iteration whether backtracking was applied, and if so, what the value of λ was. Submit the used code.

### **Solution**

Looking at the table below you can see backtracking is happening at k = 3 because lambda value is lower than 1 and at k = 4 the convergence and error gets a spike as a result.

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## **iii)**

State an estimate the accuracy after 7 iterations. The estimate must be based on the data

obtained from the 7 iterations and must be stated with a clear argument of how you computed it. Without such an argument, there is no points for the answer.

### **Solution**

The accuracy after 7 Newton iterations was estimated using

* Where ​ is the difference
* Where is the estimated convergence

These values are computed in printRoots()

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# **Exercise 3**

## **i)**

* Rewrite Eq. (1) into a set of two first order ODE's. State the two first order ODE's.

### **Solution**

To convert the equation into a set of two first order ODE’s I introduce a new variable:

This means that:

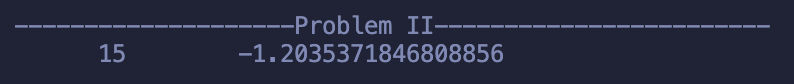
Substituting this into the equation we get the two first order ODE’s:

## **ii)**

State with at least 7 digits the value of x’’(t0). Submit the used code. HINT: You should get

X´´(t0) ≃ −1.20.

### **Solution**



## **iii)**

Use the Midpoint method with N = 80; h= 0.25 to generate a solution for x(t); −10 ≤t≤

10. State the value of x(t) at t = 10. State plots of x(t), x’(t) and XF(t)−x(t). Submit the

used code.

### **Solution**

If I knew how to save the data in a file and then maybe use python or MATLAB to read and process it I would have continued with the problem. In the code I also print out the 80 steps.

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# **Exercise 4**

## **i)**

With N−1 = 2^k ; k= 1,...,20 use the Simpson Method method to approximate the integral.

State the results in a table similar to those used during the course. Submit the used code.

### **Solution**

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## **ii)**

Use Richardson extrapolation to estimate the order at N−1 = 2^20. State the result. Submit

the used code.

### **Solution**

Can also be seen in the table above.

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## **iii)**

The estimated order is different than the expected order. State an explanation for the difference and your guess on the exact order that you have estimated.

### **Solution**

The function has square roots at the ends, so its slope changes too fast at x=0 and x=2. Simpsons method needs the function slopes to not change this way, so the error wont go down as fast. This would be why the order is around 1.5 instead of 4.

## **iv)**

State the estimated accuracy on the result at N−1 = 2^20 using the estimated order. State clearly how you compute the accuracy estimate.

### **Solution**

The estimated accuracy is computed using **Richardson extrapolation, equation 7 in Richardson.pdf from numerical methods course in itslearning resources.**

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## **v)**

State what other method could have been used to achieve high accuracy with likely much

fewer f(x) computations?

### **Solution**

The **midpoint method could be better because it** doesn’t use the values at the edges like Simpson at x=0 and x=2. Midpoint evaluates the function inside the interval.

# **Exercise 5b**

## **i)**

Consider N = 2. State an analytical expression of the semi discrete form for this problem.

State also the value of for t = 0

### **Solution**

I use the semidiscrete formula for explained in uge14pres.pdf slide 5.

I use j = 1, which is the only inner point when N=2 and find at t = 0